

## 2 Utility Maximization

### 2.1 Example

*[Ann is sitting by a table. Barbara, her sister, enters.]*

*Barbara:* Hey, what's up?

*Ann:* Nothing.

*Barbara:* But you're depressed.

*Ann:* No, I'm not.

*Barbara:* C'mon, I know you better than that. You are obviously, positively, definitely depressed.

*Ann:* I'm not depressed, it's just that . . .

*Barbara:* . . .yes?

*Ann:* Well, you won't tell anyone, will you?

*Barbara:* Of course not, you can trust me; this is what you have big sisters for.

*Ann:* The same way I could trust you then with the chocolate?

*Barbara:* Oh, don't be silly, we were kids then. *[Both smile.]*

*Ann:* Well, the thing is that I have three guys who want to date me, and I can't make up my mind.

*Barbara:* I see. Well, I have some experience in this matter. Do you like them?

*Ann:* Uh-huh.

*Barbara:* All three of them?

*Ann:* Uh-huh.

*Barbara:* You're not very selective, are you?

*Ann:* Thank you very much. Why not say, my little sister is so wonderful that she attracts the best guys around?

*Barbara:* Sure, sure, that's exactly what I meant. Anyway, you like all three?

*Ann:* Yes, sort of, you know, there are pluses and minuses, no one is perfect.

*Barbara:* Do you love any of them?

*Ann:* I don't know, I *think* so, I mean I sort of love each of them in some way.

*Barbara:* That means you're not *in* love with any of them.

*Ann:* Maybe. But I still don't want to be all alone. What happens if I'm never in love?

*Barbara:* Okay, here's my idea: you sit down, and attach to each one of them a number. The better the guy is, the higher the number. Then you select the one with the highest number.

*Ann:* That sounds crazy. Did you learn that at school?

*Barbara:* Yes, we called it utility maximization.

*Ann:* Sounds just like the kind of thing that you would study in a business school. How to maximize your utility. Great. Was the course titled "How to use and abuse your boyfriend"?

*Barbara:* Why abuse? What are you working yourself up about?

*Ann:* Just listen to your words: utility, maximization—this sounds so cold, so heartless! Do they also teach you to choose the boy who's richest or whose father is best connected?

*Barbara:* No . . . .

*Ann:* This is love we're talking about, not money! This is about people, and relationships, and emotions, not about stocks and, and . . . .  
[*She begins crying.*]

*Barbara:* Wait a minute, cool down, okay? First, they do not teach us how to choose boyfriends there; it's a business school, not a summer camp. I was just thinking about this idea because of how we make decisions. Second, I think you're carried away with rhetoric.

*Ann:* Yes, sure, if I don't think you're the greatest genius on earth, I'm carried away with rhetoric.

*Barbara:* No, I mean it, could you give me a chance to explain?

[Ann is silent, but it's clear she's willing to listen.]

Barbara: And please, without getting overexcited and without attaching meaning to the particular words—that's what I meant by rhetoric: forget about the terms, think about their contents.

Ann: OK, I'm listening. But do me a favor, and don't make it as long as last time with the derivatives. I understood nothing.

Barbara: Don't worry, this is purely about concepts. And it's short.

Ann: Okay, go ahead!

Barbara: Think of your choice between any pair of these candidates.

Ann: "Candidate"! This isn't politics!

Barbara: You see, you get all hung up on the words. What do you care if I call them *candidates* or *choices* or *guys* or *alternatives*?

Ann: It's important how you refer to people. Language has an impact on the way we think. You think of them as alternatives, and immediately I start thinking that each of them is dispensable.

Barbara: I see your point. In fact, I may even agree with you, for a change. Seriously, I think that what you just said is quite deep. I wonder if economists don't get a lot of unnecessary criticism because of a poor choice of words.

Ann: It's not unnecessary. You just agreed that language has its power.

Barbara: I meant, unnecessary in the sense that what these economists have to say is actually quite sensible, but because they often choose words that turn people off, people don't listen to what they have to say.

Ann: Okay, but I'm mature and open-minded and I'm listening.

Barbara: So: consider your choice between any pair of guys.

Ann: Any pair?

Barbara: With three guys you have exactly three pairs. With four guys you would have six pairs, with five, ten pairs, and so on.

Ann: You promised no derivatives.

Barbara: Derivatives? Derivatives have to do with calculus. This is combinatorics.

Ann: You know what I mean.

Barbara: Okay, so take these three pairs—think of  $a-b$ ,  $b-c$ ,  $a-c$ .

Ann: I'm thinking of them.

*Barbara:* Would you like to be able to choose between any two?

*Ann:* Yes, of course, that's what I'm trying to do.

*Barbara:* We call this *completeness*. It means that you can always make a choice, that your preferences are *complete*.

*Ann:* And if I find two of them just as good?

*Barbara:* Ties are allowed. You can say that you are indifferent between the two; each is as good as the other. Then you may choose the first that comes to mind, but you won't need to change your choice later on. By the way, it's good for your guys, too.

*Ann:* Huh?

*Barbara:* Otherwise you'd drive them nuts. You'd say yes and no, first you and then him, and then maybe. Do you know, for instance, that Franz Kafka was twice engaged to marry the same woman, and he canceled the marriage both times?

*Ann:* Really?

*Barbara:* Yes, it didn't really make her happy.

*Ann:* Why did he do that?

*Barbara:* Well, he was just incapable of making a decision. The point is there's nothing very romantic about this.

*Ann:* Okay, I get it.

*Barbara:* Good. Now: would you like your choices between pairs to be transitive?

*Ann:* What's that?

*Barbara:* Transitive. This means that if you think that  $a$  is at least as good as  $b$ , and  $b$  is at least as good as  $c$ , you also think that  $a$  is at least as good as  $c$ .

*Ann:* I guess so.

*Barbara:* Sure, you want to make such decisions!

*Ann:* Here we go again. Big wise sister telling Ann what she wants.

*Barbara:* No, no, no, not because I'm your big sister, and not because I'm wise, though both are true.

[*Ann rolls her eyes.*]

*Barbara:* You want to be transitive because otherwise you'll be dating  $c$  and leaving him for  $b$ , then dating  $b$  and leaving him for  $a$ , and then you'll send  $a$  away and go back to  $c$ , and so on, until they're all fed up

with you. If you are not transitive, you will be cruel to all the guys involved, and if they have any backbone, you'll be cruel to yourself, too.

*Ann:* Oh, I thought that being faithful to one means being cruel to all the others.

*Barbara:* Did I ever say that?

*Ann:* Da Ponte did, giving this line to Don Giovanni.

*Barbara:* Oh, good. I was afraid I might have been too honest.

*Ann:* Very funny.

*Barbara:* But you get the point—if you want to be neither as indecisive as Kafka nor as fickle as Don Giovanni, you have to be complete and transitive.

*Ann:* Okay, suppose I am. What wouldn't one do for one's sister!

*Barbara:* The point is that if you are complete and transitive in your preferences, then it is as if you are maximizing a utility function.

*Ann [suspiciously]:* Function? This is something with a derivative, isn't it?

*Barbara [smiling]:* It might have a derivative in calculus. But all I mean is a rule, a way to assign numbers to alternatives.

*Ann:* What's a way? What is not a way?

*Barbara:* Just think of a table, where in one column you have the name of the alternative, and in another, the numerical value you attach to it.

*Ann:* If you mean a table, why do you call it a function? Sometimes I feel you really don't want me to understand what you're saying.

*Barbara:* I'm sorry. Don't give me this look, I really mean it. The reason it's called a function is that sometimes it will not be given by a table but by a formula. You know, like writing  $2x$  instead of listing the value for each and every value of  $x$ .

*Ann:* Okay. But I can think of a function as a table?

*Barbara:* Yes, you can think of it as a table of values that is sometimes more succinctly described by a formula.

*Ann:* Great. But what did you want a function for?

*Barbara:* You're so argumentative, I nearly forgot why I mentioned a function in the first place. But I think it's coming back to me. I said that if your preferences are complete and transitive, then I can think of you as if you were maximizing a utility function.

*Ann:* As if? But I'm not.

*Barbara:* Well, this is up to you. But let's start by agreeing that this is now only a matter of representation. One can say, "Ann is choosing among her alternatives by maximizing a utility function" and one can also say, "Ann is choosing whom to date in a complete and transitive way, or a decisive and faithful way," and these two statements mean exactly the same thing. It's a mathematical theorem.

*Ann:* What is?

*Barbara:* That if you have a preference—a way to compare pairs of alternatives—that is complete and transitive, then it can be represented by a utility function, so that between any two alternatives the one with the higher utility is chosen.

*Ann:* Always?

*Barbara:* Well, at least if you have finitely many alternatives. And, pretty as you are, I think that even you don't have infinitely many suitors.

*Ann:* You're so clever.

*Barbara:* More than you'd believe. There's even more: not only can I look at you and say, "Ann is maximizing a utility function," without thinking anything bad about you, I can even tell you that finding a utility function and maximizing it is the only method I know that can guarantee that you will indeed be complete and transitive in your preferences.

*Ann:* So you seriously suggest that I assign a number—call it *utility* if this makes you happy—to each guy and choose the one with the highest number.

*Barbara:* Yes, that is precisely what I suggest.

*Ann:* But I really hate the word *utility*. It makes me think of gas, electricity, and cable TV, not of love.

*Barbara:* Can we call it *payoff*?

*Ann:* Payoff is what you get when you gamble on horses. Or when you're killed by the mafia.

*Barbara:* Call it whatever you like. I thought we agreed not to attach too much importance to names. Just assign numbers to your alternatives.

*Ann:* But I really don't know how I would do that. How do I know if Bob should have a higher number than, say, Jim?

*Barbara:* Ask yourself, which one do you like better?

*Ann:* But that's precisely the point; I don't know which one I like better!

[*Barbara is silent.*]

*Ann:* I mean, this is what you were supposed to help me sort out in the first place, weren't you?

*Barbara:* You know what? I'll think about it.

## 2.2 Two Points

The example in the previous section illustrates two main points. The first is that terms like *utility* and *maximization* should not turn you off. They do not preclude emotional decision making, love and hate, lofty or base motives. To say that someone maximizes a utility function is merely to say that she is coherent in her choices. Mother Teresa could possibly be described as maximizing the number of healthy children in the world. That is, she maximized a certain function. Adolf Hitler tried to maximize the percentage of Aryan people in Germany. He also maximized a function. Thinking of Mother Teresa and Adolf Hitler as utility maximizers only says that each of them pursued a goal in a coherent way. It does not mean that they are equivalent in terms of ethical judgments, character, or anything of the sort. You are likely to admire Mother Teresa for her utility function and to loathe Adolf Hitler for his. The notion of utility maximization leaves room for all these attitudes.

The second important point, made at the end of the dialogue, is that it doesn't always help to want to maximize a utility function. In the absence of additional structure in the problem, the mathematical equivalence mentioned in the dialogue leaves us no better off than we were when we started.

The dialogue refers to a theorem stating that comparison between pairs is complete and transitive if and only if it can be described by maximization of a function (a utility function). Appendix B provides mathematical details and two formal versions of this theorem. I now turn to the theorem's interpretations.

## 2.3 Interpretations

The theorem in appendix B has three types of interpretations. One concerns normative applications of the theory of utility maximization,